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Collective modes in tight-binding superlattices with chain sheets

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Abstract. By extending our plasma theory of superlattices on the basis of several tight-binding metallic sheets, we have derived dispersion relations for the collective excitations of a tight-binding superlattice consisting of alternating layers of one chain sheet and N identical layers. An expression for the Coulomb interaction is worked out for quasi-one dimensional systems in terms of a screened factor. The plasma frequency is obtained as a function of the wavevector analytically as well as numerically for several specific cases.

1. Introduction

Since the discovery of high T_c superconductivity [1], it has been known that oxide superconductors can be described as superlattices with a basis of several metallic sheets. Recently, Triscone and his collaborators [2] have reported the successful preparation of real superlattices of high-T_c superconductors consisting of alternating layers of 1:2:3 materials. On the other hand, it has been argued [3] that investigating collective modes in high-T_c perovskites might be a clue to understanding the mechanisms that govern the superconductivity of these materials. These stimulated us to present a plasma theory [4] of tight-binding superlattices of La-, Bi-, and Tl-based high-temperature oxide superconductors. To our knowledge, previous studies [5–10] were concerned with the elementary collective excitations (plasmons) of the two-dimensional (2D) and layered free-electron-gas models with or without chain sheets, both experimental and theoretical. However, in view of the poor conductivity of the ceramics in their normal states, free-charged-particle theories are inappropriate and we prefer rather a tight-binding description. Difficulties are encountered in the study of plasma oscillations in tight-binding systems, because the Coulomb vertex in the momentum space ceases apparently to be a function of the momentum transfer between two interacting particles alone, so that reasonable approximation schemes such as the random-phase approximation (RPA) are no longer feasible. Nevertheless, for tight-binding systems with inclusion of nearest-neighbour hopping only, we have devised a specific renormalization procedure [4, 11], which restores a simple dependence on the momentum transfer alone of a renormalized vertex, so that standard techniques such as the RPA can be readily employed as in the case of the free electron gas.

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The 1:2:3 materials such as $YBa_2Cu_3O_7$ can be treated as tight-binding superlattices with chain sheets [8–10]. This idealized model provides an adequate description of the energy band structure near the Fermi surface [12]. As a generality of [4], this paper is concerned with the plasmon dispersion relations of tight-binding superlattices consisting of one chain sheet and N identical layers in a single supercell, which is relevant with 1:2:3 materials for $N = 2$. In a future publication, we hope to understand the role of these plasmon modes in superconductivity.

2. Theory

We write the wavefunction of a carrier localized around a site x_i in a one-dimensional chain of constant a as

$$\varphi^{(1)}_{n\alpha}(x - x_i, y - y_\alpha, z - z_n) = \phi^{(1)}(x - x_i)\xi_{n\alpha}(y - y_\alpha, z - z_n) \tag{1}$$

where (n, α) labels the αth chain within the nth supercell with coordinate $y_\alpha = \alpha b$ along the b axis and $z_n = nc$ along the c axis and $\xi_{n\alpha}$ is an envelope wavefunction describing the confinement in the b and c directions. Similarly the wavefunction of a carrier localized around a site $r_i = (x_i, y_i)$ in a two-dimensional square lattice of constant $a(= b)$ is written as

$$\varphi^{(2)}_{nl}(r - r_i, z - z_{nl}) = \phi^{(2)}(r - r_i)\zeta_{nl}(z - z_{nl}) \tag{2}$$

where (n, l) labels the lth plane within the nth supercell with coordinate $z_{nl} = nc + \Sigma_{i=1}^{l} c_i$ $(l = 1, 2, \ldots, N)$, along the c axis and ζ_{nl} is an envelope wavefunction describing the confinement in the c direction. For simplicity, we assume $|\zeta_{nl}|^2$ to be delta functions and $|\xi_{n\alpha}|^2$ to be Gaussian function without loss of generality. Changing $\xi_{n\alpha}$ and ζ_{nl} to more complicated localized functions does not change the conclusion of this paper in any qualitative fashion. Therefore we can neglect the overlap of $\varphi^{(1)}_{n\alpha}$ and $\varphi^{(2)}_{nl}$ in the c direction and that of $\varphi^{(1)}_{n\alpha}$ in the b direction, respectively. A quasi-one-dimensional (quasi-1D) Bloch function $\psi^{(1)}_{n\alpha}(p_x; x, y, z)$ can accordingly be constructed in terms of $\varphi^{(1)}_{n\alpha}$ as

$$\psi^{(1)}_{n\alpha}(p_x; x, y, z) = L^{-1/2}A^{(1)}(p_x)\sum_{x_i} \varphi^{(1)}_{n\alpha}(x - x_i, y - y_\alpha, z - z_n)\exp(ip_x x_i) \tag{3}$$

and a quasi-two-dimensional Bloch function $\psi^{(2)}_{nl}(\boldsymbol{p}; \boldsymbol{r}, z)$ in terms of $\varphi^{(2)}_{nl}$ as

$$\psi^{(2)}_{nl}(\boldsymbol{p}; \boldsymbol{r}, z) = S^{-1/2}A^{(2)}(\boldsymbol{p})\sum_{r_i} \varphi^{(2)}_{nl}(\boldsymbol{r} - \boldsymbol{r}_i, z - z_{nl})\exp(i\boldsymbol{p}\cdot\boldsymbol{r}_i) \tag{4}$$

with $A^{(1)}(p_x)$ and $A^{(2)}(\boldsymbol{p})$ the respective normalization constants, L the length of conductive chains and S the area of conductive layers. In the approximation including only overlap between wavefunctions of nearest-neighbour sites, the conduction bands for chains and layers are given, respectively, by the standard method [13].

$$\varepsilon^{(1)}(p_x) = [A^{(1)}(p_x)]^2[\epsilon_0^{(1)} - 2J^{(1)}\cos(p_x a)] \tag{5}$$

$$\varepsilon^{(2)}(\boldsymbol{p}) = [A^{(2)}(\boldsymbol{p})]^2\{\epsilon_0^{(2)} - 2J^{(2)}[\cos(p_x a) + \cos(p_y a)]\} \tag{6}$$

with $\epsilon_0^{(i)}$ the relevant on-site single-particle energy and $J^{(i)}$ the relevant hopping integral between nearest-neighbour sites. The essential point of the theory is the observation that in the same approximation the matrix element of the Coulomb potential between

wavefunctions $\psi^{(i)}_{n_1 i}(p+q)$, $\psi^{(j)}_{n_2 j}(p'-q)$ and $\psi^{(j')}_{n_3 j'}(p')$, $\psi^{(i')}_{n_4 i'}(p)$ can be factorized to write [4, 11]

$$V_{n_1 i, n_2 j, n_3 j', n_4 i'}(p, p', q) = A^{(i)}(p)A^{(i)}(p+q)A^{(j)}(p')A^{(j)}(p'-q) \times \hat{T}^{\mathrm{T}}(p)\hat{V}_{n_1 i, n_2 j}(q)\hat{T}(p')\delta_{n_1 i, n_4 i'}\delta_{n_2 j, n_3 j'} \tag{7}$$

where ni denotes $n\alpha$ for $i = 1$ and nl for $i = 2$, respectively, and $\hat{T}$ is a row matrix (the superscript T means transpose):

$$\hat{T}^{\mathrm{T}}(p) = [1, \cos(p_x a), \sin(p_x a), \cos(p_y a), \sin(p_y a)] \tag{8}$$

and $\hat{\mathbf{V}}$ is a 5×5 matrix which can be factorized further into

$$\hat{V}_{ni, n'i'}(q) = \hat{p}^{(i)}(q)V_{ni, n'i'}(q)\hat{p}^{(i')\mathrm{T}}(-q) \tag{9}$$

where $\hat{p}^{(i)}$ is a column matrix with elements defined by overlap integrals between nearest-neighbouring $\phi^{(i)}$ orbitals while

$$V_{ni, n'i'}(q) = \begin{cases} (2e^2/\epsilon)[K_0(q_x\sqrt{(y_\alpha - y_{\alpha'})^2 + (z_n - z_{n'})^2}) \\ \qquad - K_0(\sqrt{q_x^2 + \beta^2}\sqrt{(y_\alpha - y_{\alpha'})^2 + (z_n - z_{n'})^2})] \\ (2\pi e^2/\epsilon q)\exp(-q|z_n - z_{n'l'}| + \mathrm{i}q_y y_\alpha) \\ (2\pi e^2/\epsilon q)\exp(-q|z_{nl} - z_{n'}| - \mathrm{i}q_y y_{\alpha'}) \\ (2\pi e^2/\epsilon q)\exp(-q|z_{nl} - z_{n'l'}|) \end{cases} \quad \text{for} \quad \begin{cases} i = i' = 1 \\ i = 1, i' = 2 \\ i = 2, i' = 1 \\ i = i' = 2 \end{cases} \tag{10}$$

where K_0 is the zeroth-order Bessel function of imaginary argument, ϵ is the background dielectric constant and β a screened factor in the direction perpendicular to the chains.

Hence by absorbing the matrix $\hat{T}$ to define a new particle propagator $\hat{G}_{ni}(p, \omega)$, which is a row matrix $\hat{G}^{\mathrm{T}}_{ni}(p, \omega) = A^{(i)}(p)G_{ni}(p, \omega)\hat{T}^{\mathrm{T}}(p)$, we are left with a reduced vertex $\hat{V}_{ni, n'i'}$ which is a function of the momentum transfer alone. The cost paid is that both the propagator and the vertex become matrix quantities. We are thus able to use the standard Feynman–Dyson technique of many-particle theory to study the effective interaction between a pair of carriers as in the case of a free electron gas. In the RPA, the effective interaction matrix is given by [4]

$$\hat{U}_{ni, n'i'}(q, \omega) = \hat{V}_{ni, n'i'}(q) + \sum_{n''i''} \hat{V}_{ni, n''i''}(q)\hat{\Pi}_{i''}(q, \omega)\hat{U}_{n''i'', n'i'}(q, \omega) \tag{11}$$

where $\hat{\Pi}_i$ is the RPA response function matrix:

$$\hat{\Pi}_i = \frac{2}{(2\pi)^d}\int \mathrm{d}^d k\, \hat{T}(k+q)[A^{(i)}(k)\,A^{(i)}(k+q)]^2 \times \frac{f(\varepsilon^{(i)}(k+q)) - f(\varepsilon^{(i)}(k))}{\varepsilon^{(i)}(k+q) - \varepsilon^{(i)}(k) + \hbar\omega + \mathrm{i}0^+}\hat{T}^{\mathrm{T}}(k). \tag{12}$$

Fourier transforming equation (11) with respect to n, n' and α, α' to make use of the periodicity of the layers in the c direction and that of the chains in the b direction, i.e.

$$\hat{U}_{ni,n'i'} = \hat{p}^{(i)}(\boldsymbol{q}) \int \frac{c}{(2\pi)^2} \mathrm{d}q_z \exp[\mathrm{i}q_z(z_n - z_{n'})]\, \hat{p}^{(i')\mathrm{T}}(-\boldsymbol{q})$$
$$\begin{cases} \int \mathrm{d}q_y U_{00}(q_x, q_y, q_z, \omega) \exp[\mathrm{i}q_y(y_\alpha - y_{\alpha'}] \\ U_{0l'}(\boldsymbol{q}, q_z, \omega) \exp(\mathrm{i}q_y y_\alpha) \\ U_{l0}(\boldsymbol{q}, q_z, \omega) \exp(-\mathrm{i}q_y y_{\alpha'}) \\ U_{ll'}(\boldsymbol{q}, q_z, \omega) \end{cases} \quad \text{for} \quad \begin{cases} i = i' = 1 \\ i = 1, i' = 2 \\ i = 2, i' = 1 \\ i = i' = 2 \end{cases} \tag{13}$$

we obtain from (11) and (9) that

$$U_{ll'} = \tilde{V}_{ll'} + \sum_{l''} \tilde{V}_{ll''} \Pi_2 U_{l''l'} \tag{14}$$

and

$$\tilde{V}_{ll'} = V_{ll'} + V_{l0}(\Pi_1 - \Pi_2) V_{0l'} / [1 - (\Pi_1 - \Pi_2) V_{00}] \tag{15}$$

for $l, l' = 0, 1, 2, \ldots, N$, In equation (15),

$$V_{00} = \frac{2e^2 a}{\epsilon} \sum_{n\alpha} [K_0(q_x \sqrt{\alpha^2 a^2 + n^2 c^2})$$
$$- K_0(\sqrt{q_x^2 + \beta^2} \sqrt{\alpha^2 a^2 + n^2 c^2}) \exp(\mathrm{i}q_y \alpha a + \mathrm{i}q_z nc) \tag{16}$$

and

$$V_{ll'} = V_{l'l}^* = \frac{2\pi e^2}{\epsilon q} \left\{ \sinh\left[q\left(c - \sum_{i=l+1}^{l'} c_i \right) \right] \right.$$
$$\left. + \sinh\left(q \sum_{i=l+1}^{l'} c_i \right) \exp\left(-\mathrm{i}q_z c \right) \right\} \Big/ [\cosh(qc) - \cos(q_z c)] \tag{17}$$

for $l + l' \neq 0$ and $l' \geq l$, where we define $\Sigma_{i=l+1}^{l'} c_i = 0$ for $l = l'$. These normalized results agree precisely with those given for the free-electron-gas models [6], except that the response functions are replaced by

$$\Pi_i(\boldsymbol{q}, \omega) = \hat{p}^{(i)\mathrm{T}}(-\boldsymbol{q}) \hat{\Pi}_i(\boldsymbol{q}, \omega) \hat{p}^{(i)}(\boldsymbol{q}) / c_i \tag{18}$$

with $c_i = a$ for $i = 1$ and $c_i = 1$ for $i = 2$. Now we should make some comments on V_{00}. If we consider the $b \to \infty$, $c \to \infty$ limit, the different chains are uncoupled, and then V_{00} describes an isolated chain. In this limit, the potential V_{00} in (16) becomes

$$V_{00}(b \to \infty, c \to \infty) = (e^2 a/\epsilon) \ln[(q_x^2 + \beta^2)/\beta]. \tag{19}$$

Its asympotic behaviour is

$$V_{00}(b \to \infty, c \to \infty) = \begin{cases} -(2e^2 a/\epsilon) \ln(q_x/\beta) \\ -(e^2 \beta^2 a/\epsilon q_x^2) \end{cases} \quad \text{for} \quad \begin{cases} q_x \to 0 \\ q_x \to \infty \end{cases}$$

which is consistent with the asymptotic behaviour given by Friesen and Bergersen [14]

for one-dimensional (1D) systems. We next consider the $c \to \infty$ limit. The different sheets are uncoupled, and then V_{00} describes a quasi-one-dimensional plane superlattice consisting of 1D chains. When $q_x \to \infty$, $V_{00} \to q_x^{-2}$. Such an asymptotic behaviour is different from that in the previous discussion [15] on quasi-1D plane superlattices, where $V_{00} \to q_x^{-1/2} \exp(-\alpha q_x)$ for $q_x \to \infty$ in the limiting case $c \to \infty$. Therefore, strictly speaking, the Coulomb interaction of the quasi-1D systems is better expressed by (16) than by previous expressions.

The condition for the existence of collective modes is given by the pole of the screened interaction. Thus plasmon modes can be obtained by solving the $(N+1) \times (N+1)$ determinant equation

$$\det|\delta_{ll'} - \tilde{V}_{ll'}\Pi_2| = 0 \tag{20}$$

and then the effective interaction between carriers in specific sheets or chains can be obtained using the Fourier transformation in equation (14).

3. Results and discussion

We now proceed to seek solutions of equation (20) for $N = 0, 1, 2$ (where N is the number of the identical layers per unit cell).

3.1. Superlattices with N = 0

For superlattices with only a single chain sheet per unit cell, equation (20) becomes

$$1 - \Pi_1 V_{00} = 0. \tag{21}$$

The RPA response function Π_1 is easily evaluated at zero temperature [8], usually with the high-frequency approximation ($\omega \gg \omega_m^{(1)} = 4J^{(1)}|\sin(q_x a/2)|$)

$$\Pi_1 = (4J^{(1)}/\pi)\{1 - [(\epsilon_0^{(1)} - \mu)/2J^{(1)}]^2\}^{1/2} q_x^2/\omega^2 \equiv A_1 q_x^2/\omega^2. \tag{22}$$

Thus, in the long-wavelength limit ($q_x, q_y \to 0$), we have

$$\omega^2 = \begin{cases} (4\pi e^2 A_1/\epsilon c)[q_x^2/(q_x^2 + q_y^2 + q_z^2)] \\ (2e^2 A_1 a/\epsilon)F(\beta)q_x^2 \end{cases} \qquad \text{for} \begin{cases} q_z c \to 0 \\ q_z c \to \pi \end{cases} \tag{23}$$

where $F(\beta)$ has the $\ln\beta$ limit in the $\beta \to \infty$ limit and is vanishingly small in the $\beta \to 0$ limit. The $q_z c \to 0$ spectrum is acoustic for $q_x^2 + q_y^2 \neq 0$ and the $q_z c = \pi$ acoustic plasmon for $q_y \to 0$ is an antisymmetric mode corresponding to the charge fluctuation of opposite sign in alternate chain sheets. Since our analysis is valid only in the high-frequency regime, we must demand that the velocity of the acoustic branch $\omega(q_z c = \pi)$ exceeds $\omega_m^{(1)}$, i.e. $\beta > \beta_c$. Using (23), we get

$$\beta_c = (\pi a/c\gamma)\exp\{(\epsilon\pi J^{(1)} a/2e^2)\sqrt{1 - [(\epsilon_0^{(1)} - \mu)/2J^{(1)}]^2}\} \tag{24}$$

with γ the Euler constant. We emphasize that the above analysis is qualitatively valid

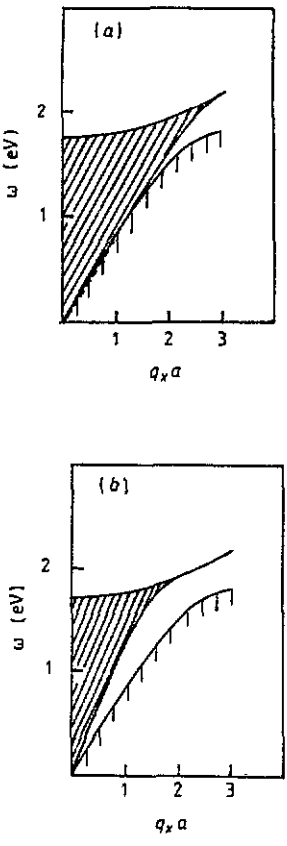

Figure 1. Plasmon modes as a function of the wavevector for the $N = 0$ case with $q_y a = 0$, and both $q_z c = 0$ and $q_z c = \pi$: (*a*) $\beta a = 5$: (*b*) $\beta a = 15$. In (*a*), the broken line denotes the $\omega(q_z c = \pi)$ mode disappearing into the single-excitation regime.

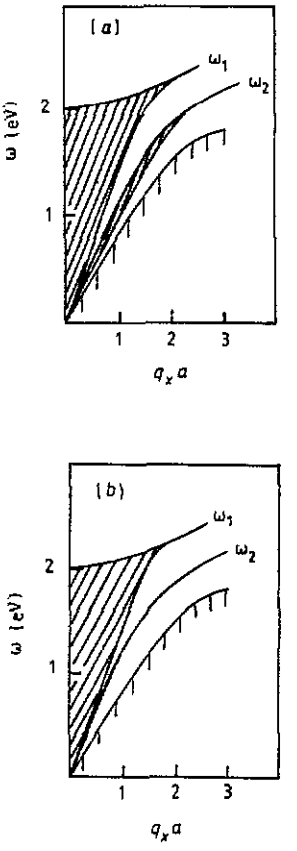

Figure 2. Plasmon modes as a function of the wavevector for the $N = 1$ case with $q_y a = 0$, and both $q_z c = 0$ and $q_z c = \pi$: (*a*) $\beta a = 5$: (*b*) $\beta a = 15$.

for the high-frequency regime. Figure 1 shows a graph of the plasmon band for $\beta a = 5$ and 15. In our calculation, for the $\phi^{(i)}$ we choose Gaussian fits to Slater wavefunctions and adopt the parameters consistent with the band calculation [12] and measured results [10] for Y–Ba–Cu–O superconductors as $a = b = 3.8$ Å, $\epsilon = 4.5$, $J^{(1)} = J^{(2)} = 0.45$ eV, $\epsilon_0^{(1)} - \mu = 0.6$ eV and $\epsilon_0^{(2)} - \mu = 0.7$ eV, which are the same for $N = 0, 1, 2$. In addition, the superlattice unit cell is chosen as $c = (N + 1)a$. As expected, the band width decreases as β increases.

3.2. Superlattices with N = 1

In the $N = 1$ case, there is a 2D layer as well as a chain sheet per unit cell. Equation (20) becomes

$$(1 - \Pi_1 V_{00})(1 - \Pi_2 V_{11}) - \Pi_1 \Pi_2 V_{01} V_{10} = 0. \tag{25}$$

The 2D RPA response function Π_2 has the following limiting forms in the high- and low-frequency regimes:

$$\Pi_2(q,\omega)=\begin{cases}(8J^{(2)}/\pi^2)[E(f)-(1-f^2)F(f)]q^2/\omega^2)\equiv A_2q^2/\omega^2 & \text{for } \omega\gg\omega_m^{(2)}\\ -(2/\pi^2)F(f)(1+i\omega/\omega_m^{(2)})\equiv -B(1+i\omega/\omega_m^{(2)}) & \text{for } \omega\ll\omega_m^{(2)}\end{cases} \tag{26}$$

where F and E are the complete elliptic integrals of the first kind and the second kind, respectively, and $\omega_m^{(2)}$ is the maximum single-particle excitation energy in the 2D layers:

$$\omega_m^{(2)}=4J^{(2)}[\,|\sin(q_xa/2)|+|\sin(q_ya/2)|\,]$$

and

$$f=\sqrt{1-[(\epsilon_0^{(2)}-\mu)/4J^{(2)}]^2}.$$

In the high-frequency regime ($\omega\gg\omega_m^{(2)}$), using (26) and (22) in (25), we get the following results. In the intermediate-coupling limit, i.e. $qc\ll 1$, for $q_zc\to 0$,

$$\omega_1^2=(4\pi e^2/\epsilon c)\{[(A_1+A_2)q_x^2+A_2q_y^2]/(q_x^2+q_y^2+q_z^2)\}. \tag{27}$$

This mode is an optical plasmon mode for $q_zc=0$, which corresponds to the in-phase motion of carriers in the layers and chain sheets. For $q_zc\to\pi$,

$$\begin{aligned}\omega_1^2&=(\pi e^2A_2c/\epsilon)q^2\\ \omega_2^2&=(2e^2A_1a/\epsilon)F(\beta)q_x^2.\end{aligned} \tag{28}$$

Clearly, the ω_1 and ω_2 modes are simply the respective 2D and 1D plasmon frequencies of the system. This conclusion is general since we have $V_{01}=V_{10}\to 0$ for $q_zc\to\pi$ and $c=2c_1$. In this case, the chain sheets and 2D layers are uncoupled. If $\omega_2<\omega_m^{(2)}$ for a specific β, this mode will be damped (to be discussed below). In the weak-coupling limit, i.e. $qa\gg 1$, one has

$$\begin{aligned}\omega_1^2&=(2\pi e^2A_1/\epsilon)q\\ \omega_2^2&=-(2e^2A_1c/\epsilon)q_x^2\ln(q_x/\beta).\end{aligned} \tag{29}$$

Here we obtain the very satisfying intuitive result that in the weak-coupling limit the ω_1 and ω_2 modes are simply the respective 2D and 1D modes.

Now we consider the low-frequency regime ($\omega_m^{(1)}\ll\omega\ll\omega_m^{(2)}$). From (25) we find that there could be just one solution satisfying $\omega_m^{(1)}\ll\omega\ll\omega_m^{(2)}$. This solution is necessarily complex (indicating the mode to be damped), since Π_2 has an imaginary part due to single-particle excitations. In the intermediate-coupling limit ($qc\ll 1$), the long-wavelength behaviour of the collective modes is given by (where γ_ω is the imaginary part of the frequency)

$$\begin{aligned}&\omega_2^2=(A_1/B)q_x && \text{for } q_zc\to 0\\ &\gamma_{\omega_2}=(A_1/4J^{(2)}aB)(q_x^2/\sqrt{q_x^2+q_y^2}) && \\ &\omega_2^2=(2e^2A_1a/\epsilon)F(\beta)q_x^2 && \text{for } q_zc\to\pi\\ &\gamma_{\omega_2}=(\pi e^2c/4J^{(2)}a\epsilon)[q_x^2\cos^2(q_zc/2)/\sqrt{q_x^2+q_y^2}] &&\end{aligned} \tag{30}$$

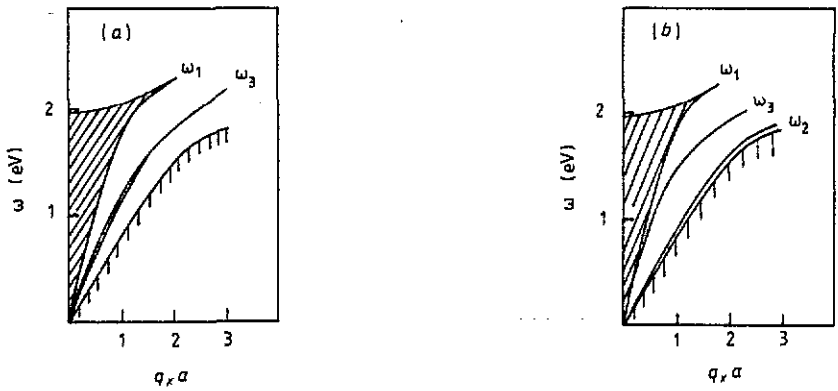

Figure 3. Plasmon modes as a function of the wavevector for the $N = 2$ case with $q_y a = 0$, and both $q_z c = 0$ and $q_z c = \pi$: (*a*) $\beta a = 5$: (*b*) $\beta a = 15$. These results should describe the superlattices of the 1:2:3 materials.

The $q_z c \to 0$ mode is one of the mixed modes (another is the mode given by (27)), and the $q_z c \to \pi$ mode is simply the 1D mode but it is damped except at $q_z c = \pi$. In the weak-coupling limit, we obtain the same result for ω_2 as (29) and the imaginary part $\gamma_{\omega_2} \simeq 0$. Thus, even though these acoustic branches lie below $\omega_m^{(2)}$, they are still approximately undamped modes in the weak-coupling limit. In figure 2 we show the plasmon band structure for the $N = 1$ case with $q_y a = 0$ (thus $\omega_m^{(1)} = \omega_m^{(2)}$ for $J^{(1)} = J^{(2)}$). Clearly, the plasmon bands are sensitive to the screened factor β. For small β, there is only one undamped band. As β increases, the number of modes increases. For a specific β, there are two plasmon bands. The ω_1 band depends on q_z while the ω_2 band for a specific β loses all dependence on q_z.

3.3. Superlattices with N = 2

We finally turn to the case of a trilayer per unit cell composed of two identical planes and a chain sheet. The free-particle system for the $N = 2$ case was previously studied by Mahan and Wu [8] and Griffin [9]. For the tight-binding case, the plasmon modes are given from (20) by

$$(1 - \Pi_2 \tilde{V}_{11})(1 - \Pi_2 \tilde{V}_{22}) - \Pi_2^2 \tilde{V}_{12} \tilde{V}_{21} = 0. \tag{31}$$

The numerical solution of equation (31) is shown in figure 3. As expected, there are three plasmon bands. The bands ω_1 and ω_2 are analogous to modes ω_1 and ω_2 in figure 2. However, in contrast with figure 2(*a*), in figure 3(*a*) the band ω_2 disappears into the single-excitation regime at $\beta a = 5$. The band ω_3 is the new feature of the $N = 2$ superlattice. This case is relevant to 1:2:3 materials, which we leave for future discussion [16]. There the effective interaction between a pair of particles in the same plane has been analysed. It can be shown that a net attraction results for superlattices with chain sheets ($N \geq 2$) in the same way as for superlattices without chain sheets [4]. The attraction arises predominantly through exchange of momentum and energy via virtually excited plasmons in neighbouring planes, which is crucial for Cooper pairing.

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